titcmt-95-1,adap-org/9501003

A set of hard spheres with tangential inelastic collision as a model of granular matter: $1/f^\alpha$ fluctuation, non-Gaussian distribution, and convective motion

Y-h. Taguchi*
Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152, Japan

Hideki Takayasu
Graduate School of Information Sciences, Tohoku University, Aoba-ku, Sendai 980, Japan

(February 5, 2008)

Abstract

A set of hard spheres with tangential inelastic collision is found to reproduce observations of real and numerical granular matter. After time is scaled so as to cancel energy dissipation due to inelastic collisions out, inelastically colliding hard spheres in two dimensional space come to have $1/f^\alpha$ fluctuation of total energy, non-Gaussian distribution of displacement vectors, and convective motion of spheres, which hard spheres with elastic collision, a conventional model of granular matter, cannot reproduce.

83.70.Fn, 52.25.Gj, 47.27.-i, 51.30.+i

Granular matter attracted attention of physicists recently [1]. It exhibits many strange behaviors: segregation [2], surface standing wave [3], bubbling [4], $1/f^\alpha$ fluctuation [5], convection [6], non-Gaussian distribution [7,8], and turbulence [7,9]. Although numerical simulations easily reproduce most of them, we cannot understand their mechanisms. This is because we do not have any universal model which can exhibit these strange behaviors observed in dynamics of granular matter. The lack of universal model prevents us from going beyond researching individual phenomena.

One of granular models used most frequently, other than conventional continuous body approximations [4,10], is a set of hard spheres colliding with each other (kinetic theory [11]). In kinetic theory, a set of hard spheres is assumed to be in thermal equilibrium, thus probability distribution function (PDF) of hard spheres' velocity should be Gaussian with variance proportional to temperature. Due to this analogy, PDF which granular particle velocity obeys is also assumed to be Gaussian, although none observed it directly, and kinetic energy of granular particle is called 'granular temperature'. However, we do not think that this analogy is suitable one for dense granular state, which corresponds to weakly fluidized granular matter; e.g., vibrating bed of powder, gas fluidized bed, chute flow, and hopper flow.

First, probability distribution function of particle velocity deviates from Gaussian [7,8]. Second, kinetic theory cannot explain the frequent appearance of $1/f^\alpha$ fluctuations [5]. Third, it cannot also explain why fluidized granular matter behaves like fluid. Cooperative macroscopic fluid mode should appear only in the limit that the number of particles is large when we employ kinetic theory. However, fluid like motion can appear in granular matter composed of less than a thousand particles [6].

The purpose of this letter is to modify hard sphere model so as to improve insufficiencies mentioned above. We show that considering tangential inelastic collision and scaling velocity give rise to the appearance of $1/f^\alpha$ fluctuations, which also explains why non-Gaussian PDF can appear. Furthermore, tangential friction allows a set of hard spheres to have cooperative modes of fluid motion, even if the number of spheres is a few hundreds.

Let us consider why kinetic theory cannot describe dense granular state. It is a natural assumption that each particle is a hard sphere. Main difference between a set of hard spheres and granular matter is whether the energy is conserved or not. The application of kinetic theory to granular matter is supported by the assumption that granular matter can control amount of dissipation so as to cancel out energy input and can converge to steady state. In order to know whether this assumption really stands, we investigate a set of hard spheres with inelastic collision in the following.

First, let us reconsider how to get steady state. Although one may believe hard spheres with inelastic collision cannot have steady state without energy input, it is not true from the theoretical point of view. If we rescale velocity properly, there can exist steady state. For example, let us consider a hard sphere moves along segment of length L. The collisions between the sphere and ends are inelastic (the coefficient of restitution $e < 1$). Suppose that the sphere is launched from one end A towards the other end B at time $t = 0$ with the velocity v. Time T when the sphere reaches the end B is L/v. After the sphere collides with the end B, the velocity v decreases to ev, and the sphere reflected by the end B will come back, at time $L/v + L/ev$, to the end A. Hence until the sphere loses its velocity completely it takes $L/v + L/ev + L/e^2v + \cdots$ which diverges, since $e < 1$. Therefore energy of the sphere

FIG. 1. Schematic of numerical setup. Bold rectangular represents dense state with $l_x = l_y = 6$. Broken rectangular indicates expanded state with $\phi \simeq 1.3$.

never vanish within finite period. What we observe here is only slowing down of motion. Rescaling time by the factor of e every time the sphere collides with one of the ends, we can have steady state: a sphere moves with constant velocity v. In the following, we regard this scaled state as a model of granular matter instead of employing kinetic theory. In this scaled state number of collisions n_{col} is employed as a unit of time.

The scaled state can be defined even in higher spatial dimensions. First, we pack hard spheres with inelastic collision into a d-dimensional box and calculate mean energy dissipation rate $\varepsilon = -\langle dE/dn_{col} \rangle$, where E is total kinetic energy and average is taken over long period. Dealing with $v \exp(\sqrt{\varepsilon} n_{col})$ instead of v, we can have scaled state in higher spatial dimensions. In the following, we describe physical variables in this scaled state by the character with tilde. For example, energy in scaled state is $\tilde{E} = E \exp(\varepsilon n_{col})$, and velocity is $\tilde{v} = v \exp(\sqrt{\varepsilon} n_{col})$.

In the following scaled state in 2D, in which we use circles instead of sphere to represent each particle, is considered. Here we consider only inelasticity of collision along tangential direction. (Here normal vector when they collide is defined as component along line passing through centers of two colliding circles, and tangential vector is perpendicular to it.) Normal components of velocity of each particle simply changes its direction oppositely due to collision. This is because we would like to make the model as simple as possible. When this definition is too simple to reproduce behaviors which we would like to consider, additional mechanism, e.g., inelastically of collision along normal direction, may have to be taken into account.

For simulating 2D system, we put hard spheres into a triangular box with elastic wall. By changing area of rectangular, we control averaged density of hard spheres. The rectangular has the width of $(2l_x + 1)a\phi$ and the hight of $2\sqrt{3}(l_y - 1)a\phi$, where $a(= 1.0)$ is radius of each circle and ϕ is expansion rate. When $\phi = 1$, granular matter forms tight rectangular lattice (See Fig.1). Thus it quantizes how much the rectangular expands comparing with dense packed state. Hereafter, y axis is taken to be parallel to vertical direction and x is horizontal. Starting from initial condition that center of each circle forms uniformly elongated triangular lattice and velocities v_x and v_y are taken from uniform random number $\in [-0.5, 0.5]$, we simulate the system composed of 256 circles ($l_x = l_y = 16$, tangential coefficient of restitution $e = 0.5$, and $\phi = 1.1$). Figure 2 shows the time development of total energy $E = \sum_i (v_{ix}^2 + v_{iy}^2)/2$, where i denote the numbering of circles. E monotonically decreases exponentially as a function of n_{col}. Using the data, we compute energy dissipation rate $\varepsilon = -\frac{1}{n_{col}^{tot}} \sum_{n_{col}} \log[E(n_{col}+1)/E(n_{col})]$, where n_{col}^{tot} is total observation time $(= 40 \times 256)$,

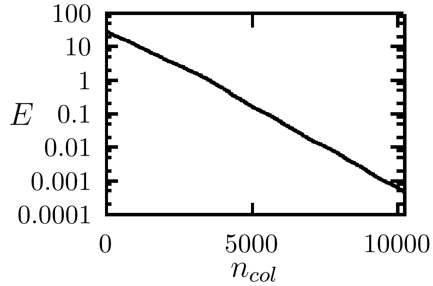

FIG. 2. Time development of energy E

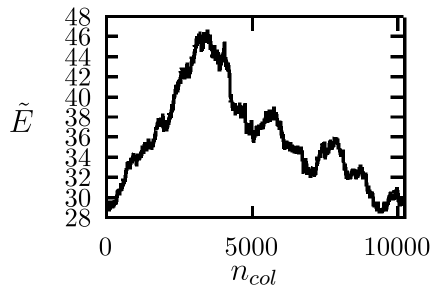

FIG. 3. Scaled energy $\tilde{E}$ in ideal granular matter

and plot scaled energy $\tilde{E} = E \exp(\varepsilon n_{col})$ in Fig.3. $\tilde{E}$ fluctuate violently, and the amount of fluctuation is not ignorable comparing with mean value $\langle \tilde{E} \rangle$. The Fourier power spectrum of $\tilde{E}(n_{col})$ is shown in Fig.4. It has power law dependence upon frequency f, and the exponent is close to -2. Thus, $\tilde{E}$ fluctuates as random walker does (i.e., the fluctuation is not stationary.). This gives rise to the conclusion that scaled state is not stationary. Thus introduction of dissipation destroys basic assumption of kinetic theory, stationarity of granular matter.

Now let us consider real granular matter. In real granular matter, energy dissipation is believed to be balanced with energy inputs, e.g., gravity, shear force, or drug force from fluid. However, these energy inputs are usually stationary so that they cannot suppress the instationary fluctuation completely. This means, even in real granular matter, we will observe the instationary fluctuation observed in scaled state. Perhaps this is the reason why we frequently observe $1/f^\alpha$ fluctuation in granular matter over the wide range of phenomena.

Next we consider deviation of velocity PDF from Gaussian. Figure 5 shows PDF of scaled velocity $\tilde{v}_x$. It is calculated from the system having 256 circles and averaged over 40 snap shots taken every 256(=number of circles) collisions. It deviates from Gaussian, and its form reminds us non-Gaussian PDF observed in numerical simulation of vibrating bed of powder. In contrast to this, a set of elastic spheres (i.e.,$e = 1$) simulated by our code gives us Gaussian PDF of velocity.

Here we can explain why scaled state can have non-Gaussian PDF. As shown above, the fluctuation of scaled energy $\tilde{E}$ behaves as fluctuation of the position of random walkers whose

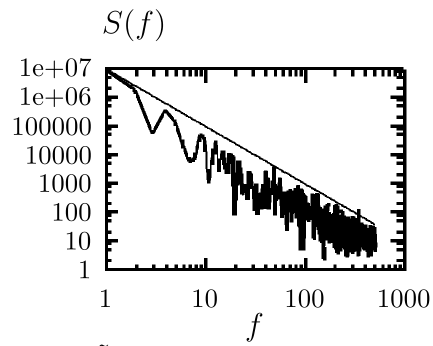

FIG. 4. Fourier power spectrum of $\tilde{E}(n_{col})$. Data points are sampled each ten collisions from time sequential data, (thus data length $= 256 \times 40/10 = 1024$) and power spectrums are averaged over obtained ten samples. Straight line indicates $1/f^2$ dependence upon frequency f

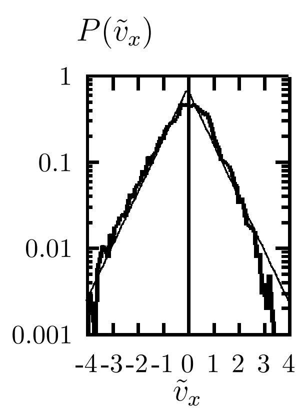

FIG. 5. PDF of scaled velocity $\tilde{v}_x$. Thin line is exponential distribution.

FIG. 6. Spatial structure of displacement vectors $\Delta \mathbf{x}_i^{\delta n}(n)$

motion is unbounded. This means, energy fluctuation is unbounded, thus, the fluctuation of velocity is not bounded, either. It prevents scaled state from having PDF of velocity with finite variance which Gaussian PDF must have. Therefore PDF of velocity in scaled state cannot obey Gaussian. It is very suggestive that exponent of power PDF observed in numerical simulation of vibrated bed of powder is very close to marginal value -3, the boundary between PDF with finite variance and that with infinite variance. (When PDF of v obeys power law PDF as $P(v) \sim v^{-\beta}$, finite variance can exist only when $\beta > 3$.) This explanation also coincides with the results obtained by Hayakawa and Ichiki [7], who found deviation of particle velocity PDF from Gaussian accompanied with $1/f^2$ fluctuation in numerical fluidized bed.

Finally we check whether scaled state can exhibit the cooperative dynamics. When we consider instantaneous scaled velocity, we could not find any spatial structures. However, when we consider displacement vectors over long period $\Delta \mathbf{x}_i^{\delta n}(n) = \mathbf{x}_i(n + \delta n) - \mathbf{x}_i(n)$, we can observe cooperative spatial structure. Figure 6 shows $\Delta \mathbf{x}_i^{\delta n}$ with $\delta n = 20 \times 256$. We can see eddy like structure, which may give rise to convection observed in real vibrated bed of powder. Thus, cooperative motion can be observed only when motion is coarse grained during long period. The reason why we observed eddy even in instantaneous velocity using soft-core potential model [9] can be understood in this consequence. In the soft-core potential model, each collision should be regarded as not individual collisions but coarse grained collisions. This interpretation of soft-core potential model is supported by recent findings where convection disappears in the limit of zero collision time in soft-core potential model [12]. The zero collision time limit in soft-core potential model clearly does not converge to the realistic situation. One should not take zero collision time limit since each collision in soft-core potential model should be regarded as a set of several collisions in real granular matters.

Finally let us discuss hierarchical structure of scaled state. Consider a granular particle surrounded by six neighboring particles fixed on the lattice points of triangular lattice (Fig.7). The motion of central movable particle is nothing but that of dispersing billiard [13]. The dispersing billiard is known to be chaotic and Ergodic. Thus, we can expect short-time motion of individual particles is chaotic, although this analogy is not exact since in granular material surrounding particles can move and collide with each other inelastically. However, we found that Fourier power spectrum obtained from time sequential data of scaled velocity $\tilde{v}_i$ obeys Lorentzian. This means, auto correlation of scaled velocity of individual particle decays exponentially as time proceeds. This decay indicates that dynamics in scaled state

FIG. 7. A granular particle as a dispersing billiard. Surrounding six particles are fixed. Broken curve show the region where center of shaded particle can move around.

still remains chaotic. Furthermore, we cannot distinguish Fourier power spectrum observed in system with elastic collision from that with inelastic collision. Thus, introducing inelastic collision does not seem to suppress chaotic motion of individual particle.

In contrast to chaotic motion of each particle, averaged quantity has long ranged or long time correlations. Scaled energy $\tilde{E}$, which is a sort of spatial averaged velocity, has long time correlation, and displacement vector $\Delta\mathbf{x}_i^{\delta n}$, which is temporal averaged velocity, has long range correlation. Thus, we can conclude that macroscopic and/or long time behaviors originates in averaging procedure. What Goldhirsch and Zanetti found [14] may support this conclusion since they have found long range correlation in inelastically colliding hard spheres after the local velocity was coarse-grained.

Finally, one should note that we do not intend to insist that scaled state can exhibit all behaviors which real granular material shows. Our modeling corresponds to ideal gas which explains equation of state correctly, but reproduction of phase transition needs van der Waals correction to it. Our modeling may also need to be modified to explain other behaviors which are not explained in this letter.

In summary, we have proposed a set of hard spheres with tangential inelastic collision as a model of dynamics of granular material. Scaled state of a set of hard spheres exhibits $1/f^{\alpha}$ fluctuation, non-Gaussian PDF, and eddy like flow, all of which are observed numerically and/or experimentally in real granular materials. Using analogy to the scaled state, we suggest that there will be no stationary state in dynamical state of granular materials, and macroscopic motion comes from averaging the chaotic motion of individual particles. HT thanks Dr. M. Takayasu for helpful discussions. YHT thanks people who developed Linux on which all calculations were performed.

REFERENCES

* Electronic address: ytaguchi@cc.titech.ac.jp

[1] For reviews and books by physicists, see e.g., H. M. Jaeger and S.R. Nagel, Science **255** 1523 (1992) ; Y-h. Taguchi, H. Hayakawa, S. Sasa and H. Nishimori (eds.), *Dynamics of Powder Systems*, Int. J. Mod. Phys. B **7** No. 9 & 10 (1993); D. Bideau and A. Hansen (eds.), *Disorder and Granular Media*, (North-Holland, Amsterdam, 1993); C. Thornton (ed.), *Powders and Grains 93*, (Balkema, Rotterdam, 1993); A. Mehta (ed.) *Granular Matter*, (Springer, New York, 1993).

[2] J.B. Knight, H.M. Jaeger, and S.R.Nagel, Phys. Rev. Lett., **70** 3728 (1993); J. Duran, T. Mazozi, E. Clement, and J. Rajchenbach, to be published in Phys. Rev. E; R. Jullien, P. Meakin, and A. Pavlovitch, Phys. Rev. Lett. **69** 640 (1992); M. Nakagawa, Chem. Eng. Sci. **49** 2540 (1994); O. Zik, D. Levine, S. G. Lipson, S. Shtrikman, and J. Stavans, Phys. Rev. Lett. **73** 644 (1994).

[3] H. K. Pak and R. P. Behringer, Phys. Rev. Lett. **71** 1832 (1993); F. Melo, P. Umbanhower, and H.L.Swinney, Phys. Rev. Lett. **72** 172 (1994).

[4] H.K Pak, P.R. Behringer, Nature **371** 231 (1994); S.Sasa and H.Hayakawa: Europhys.Lett. **17** (1992)685; T.S. Komatsu and H. Hayakawa, Phys. Lett. A **183** 56 (1993).

[5] T. Pöschel, J. Phys. I **4** 499 (1994); G. Peng, and H.J.Herrmann, Phys. Rev. E **49** R1796 (1994); M. Matsushita, S. Horikawa, and A. Nakahara, private communication.

[6] Y-h. Taguchi, Phys. Rev. Lett. **69** 1367 (1992); J. A. C. Gallas, H. J. Herrmann, and S. Sokołowski, *ibid* 1371; P. Evesque and J. Rajchenbach, Phys. Rev. Lett. **62** 44 (1989); C. Laroche, S. Douady, and S. Fauve, J. Phys. (France) **50** (1989) 699.

[7] K. Ichiki and H. Hayakawa, preprint.

[8] Y-h. Taguchi and H. Takayasu, preprint (chao-dyn/9406012); S. Warr, G. T. H. Jacques, and J. M. Huntley, Powder Technology **81** 41 (1994); Y-h. Taguchi in *Proc. Numerical Methods in Multiphase Flows*, edited by C. T. Crowe et al. **FED-185** (ASME, New York, 1994) p. 251.

[9] Y-h. Taguchi, J. Phys. II (France) **2** 2103 (1992); Europhys.Lett. **24** 203 (1993); Fractals **1** 1080 (1993); to be published in Physica D (chao-dyn/9405003).

[10] D.C. Hong, S. Yue, J.K. Rudra, M.Y. Choi and Y.W. Kim, Phys. Rev. E **50** 4123 (1994); J. Lee, and M. Leibig, J. Phys. (Paris) I **4** 507 (1994).

[11] C. S. Campbell, Annu. Rev. Fluid Mech. **22** 57 (1990); D. Gidaspow, *Multiphase Flow and Fluidization* (Academic, Boston, 1994).

[12] S. Luding, E. Clement, A. Blumen, J. Rajchenbach and J.Duran, Phys. Rev. E **50** R1762 (1994); *ibid* 4113.

[13] Ya. G. Sinai, Russ. Math. Surv. **25** 137 (1970).

[14] I. Goldhirsch and G. Zanetti, Phys. Rev. Lett. **70** 1619 (1993).